

Rotational Motion — Complete Formula Card

- Rotation is translation with $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$ — every linear formula has a spin twin
- The shape number I/MR^2 decides every rolling race: sphere beats disc beats ring, always
- Equilibrium needs BOTH $\Sigma F = 0$ and $\Sigma \tau = 0$ — one without the other is not standing still

What	Formula	Remember
Balance point	$\Sigma m_i x_i / M$	moves as if all mass were there; inside forces can't shift it
Turning power	$\tau = \text{force} \times \text{distance} \times \sin\theta$	through the pivot = zero
Spin laziness	$I = \Sigma mr^2$	ring MR^2 , disc $\frac{1}{2}MR^2$, rod $ML^2/12$, sphere $\frac{2}{5}MR^2$
Axis shift	$I = I_{\text{bal}} + Md^2$	balance-point axis is smallest
Spin Newton	$\tau = I\alpha$	fixed axis or balance-point axis
No-slip bridge	$v = R\omega$, $a = R\alpha$	string/wheel grip
Spin quantity	$L = I\omega$ / mvr	no outside turning = locked
Rolling energy	$\frac{1}{2}Mv^2(1 + I/MR^2)$	shape number: sphere 1.4, disc 1.5, ring 2.0
Ramp race	$a = g \sin\theta / (1 + I/MR^2)$	mass & size cancel
Spin energy	$\frac{1}{2}I\omega^2$; power = $\tau\omega$	rpm $\times 2\pi/60$ first!
Standing still	$\Sigma F = 0$ and $\Sigma \tau = 0$	turning about supports kills unknowns
Topple angle	$\tan\theta = \text{half-base} \div \text{balance height}$	vs slide at $\tan\theta = \mu$

Mistakes to avoid

- Using I about the wrong axis: shift with $I = I_{\text{bal}} + Md^2$ before anything else
- Rolling KE forgotten: a rolling body carries $\frac{1}{2}I\omega^2$ on top of $\frac{1}{2}Mv^2$
- rpm treated as rad/s: convert rpm $\times 2\pi/60$ first — the classic unit trap
- Toppling vs sliding confusion: compare $\tan\theta$ with μ — whichever threshold is hit first wins